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FRACTURE EMANATING FROM STRESS CONCENTRATORS IN MATERIALS: LINKS WITH CLASSICAL FRACTURE MECHANICS

PRELOM ZARADI KONCENTRATORJEV NAPETOSTI V MATERIALIH: POVEZAVA S KLASIČNO MEHANIKO LOMA

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Fracture emanating from stress concentrators is characterised by the fact that the critical level for a structure exhibiting a defect is less than those corresponding to the net limit stress. Physically, this effect means that the presence of a defect is worse than the simple reduction of the section bearing the applied load. Any defect tip can be considered as a notch of radius ρ, angle ψ and length a. The critical notch stress intensity factor can be used in fracture toughness. This parameter is helpful for measuring the fracture toughness of very brittle materials like ceramics or glass for which precracking is practically impossible. It is necessary to point out that if the notch angle is not zero, the fracture toughness has the units MPa m^{α}. For elasto-plastic fracture emanating from a notch, several approaches are possible in terms of stress), of strain or of strain-energy density or energy. These possibilities are as follows:
– the above-mentioned critical Notch Stress Intensity Factor can be considered as a global stress fracture criterion.
– the critical energetic parameter J_{lc},
– the Notch Ductility Factor is a local strain fracture criterion for elasto-plastic material. It is connected to the notch strain distribution.

Key words: notch effect, local fracture criterion, energetic fracture criteria, strain density

Za prelom, ki je posledica koncentratorjev napetosti, je značilno, da je kritični nivo pri strukturi z napako manjši od čiste mejne napetosti. Fizikalno to pomeni, da je prisotnost napake bolj škodljiva kot zmanjšanje nosilnega prereza, ki nosi zunanjo obremenitev. Vsako napako lahko predstavimo kot zarezo s polmerom ρ, kotom ψ in neko dolžino a. Pri žilavosti loma uporabljamo kritični faktor intenzitete napetosti. Ta parameter se uporablja za določevanje žilavosti loma zelo krhkih materialov, npr. keramike in stekla, za katera je nemogoče napraviti začetno razpoko. Če je zarezni kot nič, ima žilavost loma dimenzijo MPa m^{α}. Za elastoplastični prelom, ki izhaja iz zareze, je mogoče uporabiti več načinov, npr. napetost, deformacijo, gostoto deformacijske energije ali energijo. Možnosti so naslednje:
– lahko upoštevamo faktor kritične zarezne napetosti kot merilo globalnega napetostnega preloma
– lahko upoštevamo energijski parameter J_{lc}
– faktor zarezne duktilnosti je merilo za lokalni deformacijski prelom za elastoplastični material. Povezan je s porazdelitvijo zarezne deformacije.

Ključne besede: vpliv zareze, lokalno merilo preloma, energijsko merilo preloma, gostota deformacije

1 INTRODUCTION

The notch effect on fracture is characterised by the fact that the critical net stress acting on the ligament area below the notch is less than the ultimate strength of the material. This effect is very sensitive to the notch geometry, which can be described by three parameters: the notch angle, the notch radius and the notch length. It is a maximum for a crack (which is a particular case of a notch with a notch radius and a notch angle equal to zero) in a structure made with an elastic material.

The stress distribution at the notch root in the elasto-plastic case can be divided into four zones (1):

- Zone I: very close to the notch tip, the stress distribution increases to the maximum stress.
- Zone II: a transition between zone I and III.
- Zone III: in this zone the stress distribution decreases as a power function of the distance.

By analogy to a purely elastic crack-tip stress distribution, this part can be considered as a pseudo stress singularity. The stress distribution can be represented by the following relationship:

$$\sigma_{yy} = \frac{K_\rho}{(2\pi r)^{\alpha}} \tag{1}$$

where K_ρ is the so-called notch stress intensity factor and α is a power exponent. The limit between zone II and zone III is called the effective distance X_{ef}, and it has been shown that it corresponds to the limit of the fracture process zone (2).

- Zone IV: far from the notch tip, the stress value tends to reach the net stress value.

The mechanism of fracture emanating from the notch or crack is fundamentally different from the traditional "hot spot" approach (i.e. fracture occurs at the point of maximum stress). It is well known that it needs a fracture process volume. In this volume the effective stress or fracture stress can be considered as an average stress that takes into account the stress distribution and the stress gradient. This approach is called the "fracture

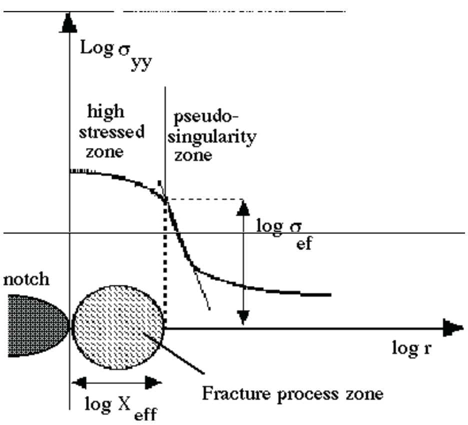

Figure 1: Schematic presentation of a local stress criterion for fracture emanating from notches
Slika 1: Shema lokalnega napetostnega merila za razpoko, ki izhaja iz zarez

volumetric approach" and can be used with stress energetic or strain parameters.

2 THE LOCAL FRACTURE CRITERION FOR FRACTURE EMANATING FROM NOTCHES

In order to get the fracture effective stress we have to take into account the stress value and the stress gradient in the neighbourhood of any point in the fracture process volume. This volume is assumed to be quasi-cylindrical, by analogy with the notch plastic zone, which has a similar shape. The diameter of this cylinder is called the effective distance. The stress value at any point inside the process zone is weighted in order to take into account the distance from the notch tip and the relative stress gradient. The fracture stress can be estimated from some average value of the weighted stresses.

This leads to a local stress fracture criterion with two parameters: the effective distance X_{ef} and the effective stress σ_{ef}. A graphical representation of this local stress fracture criterion is provided in **Figure 1**, where the logarithm of the stress normal to the notch plane is plotted versus the logarithm of distance, the effective stress and distance are presented. A graphical procedure for determining X_{ef} has been proposed by [2]. It has been shown that the effective distance is connected to the minimum of the relative stress gradient χ, defined by:

$$\chi = \frac{1}{\sigma_{yy}} \cdot \frac{d\sigma_{yy}}{dr} \tag{2}$$

The effective stress is defined as the average of the weighted stress inside the fracture process zone:

$$\sigma_{ef} = \frac{1}{X_{ef}} \int_0^{X_{ef}} \hat{\sigma}_{ij} \, dx \tag{3}$$

where the weighted stress is given by:

$$\hat{\sigma}_{ij} = \sigma_{ij} \cdot (1 - \chi x) \tag{4}$$

The fracture criterion is a two-parameter criterion. For a CrMoV steel with a fine carbide (FC) microstructure (Yield stress 771 MPa), the mean value of the effective stress is 1223 MPa, which can be compared to the average maximum local stress at a fracture σ_{max} of 1310 MPa. The critical notch stress intensity factor can be used as a measure of fracture toughness, as can be seen for Float Glass in **Figure 2**.

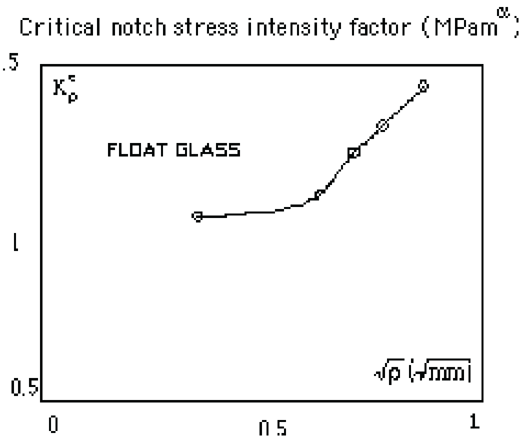

Figure 2: Evolution of the critical notch stress intensity factor versus the square root of the notch radius for Float Glass
Slika 2: Odvisnost med kritično zarezno napetostjo in korenom polmera zareze za steklo

3 ENERGETIC FRACTURE CRITERIA FOR NOTCHED COMPONENTS

3.1 Influence of the notch radius on the critical value of the energetic parameter J

For non-linear behaviour, two energy-based fracture criteria can be used: the critical non-linear energy release rate of Liebowitz [3] (equ. 5) and the energy parameter J of Turner [4] (equ. 6).

$$\widetilde{G}_{Ic} = -\frac{1}{B} \cdot \frac{\partial U_{nl}}{\partial a} \tag{5}$$

$$J_{Ic} = \frac{\eta U_{nl,c}}{B \cdot b} \tag{6}$$

with $b = W\text{-}a$ and $U_{nl,c}$ equal to the non-linear work done during fracture. Assuming that $J_{Ic} = \widetilde{G}_{Ic}$ we have:

$$\eta = -\frac{\partial LnU_{nl}}{\partial a} \tag{7}$$

From this formula we get the evolution of η, as a function of depth and notch root radius:

$$\eta = \eta(a, \rho) \tag{8}$$

The evolution of the η factor as a function of notch root radius shows an absolute minimum with the abscissa ρ_c (**Figure 3**). Similarly, we notice that for radius values below ρ_c, the η factor decreases linearly with ρ. Beyond this critical abscissa, the η factor increases with ρ and becomes approximately constant

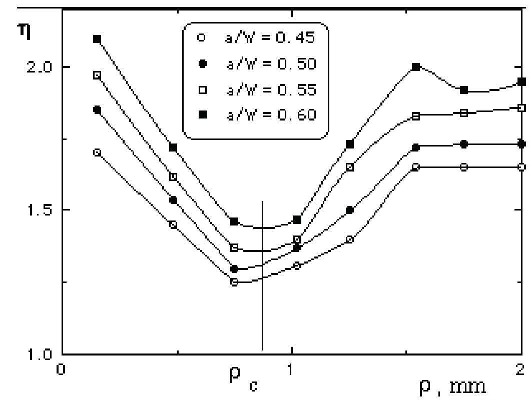

Figure 3: Evolution of η with notch radius for a constant relative depth Charpy U notch specimen
Slika 3: Evolucija η s polmerom zareze za konstantno relativno globino Charpyjevih preizkušancev z U-zarezo

for a radius ranging between 1.54 and 2 mm. The difference between the η factor for cracks and notches with the same length can reach 36 % of the latter, which is important and justifies the present approach.

In the following we will call the fracture toughness of the notched specimen the "apparent fracture toughness", written $J_{\mathrm{Ic,App}}$. J_{Ic} is only defined for cracked specimens.

$$J_{\mathrm{Ic,App}} = \frac{\eta(\alpha,\rho)U_{\mathrm{nl,c}}(\alpha,\rho)}{B \cdot b} \tag{9}$$

An example of the evolution of the apparent fracture toughness versus notch radius is given in **Figure 4**. We can see that for radii less than a critical value, ρ has no influence on the fracture toughness, but for radii greater than this critical value $J_{\mathrm{Ic,App}}$ increases linearly.

3.2 Strain energy density at the notch tip

If we plot the strain-energy density versus the notch-tip distance in a bilogarithmic graph, we get the distribution presented in **Figure 5**.

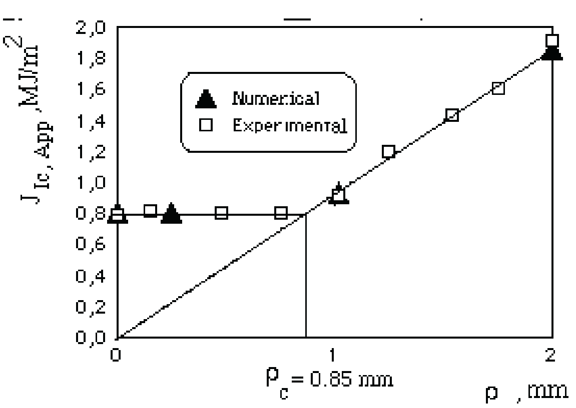

Figure 4: $J_{\mathrm{Ic,app}}$ versus notch radius for a Charpy U notch specimen in steel
Slika 4: $J_{\mathrm{Ic,app}}$ v odvisnosti od polmera zareze za jeklene Charpyjeve U-preizkušance

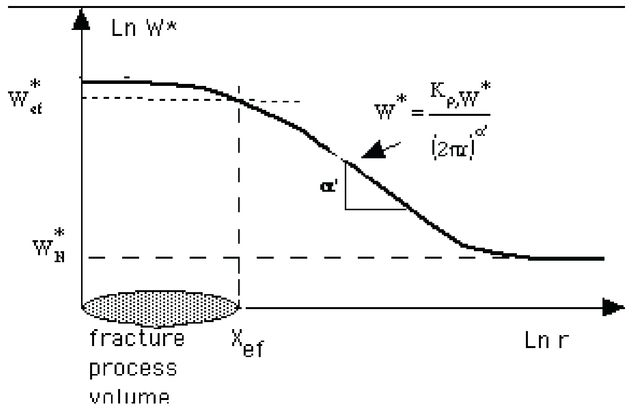

Figure 5: Distribution of the strain-energy density at the notch tip
Slika 5: Porazdelitev gostote deformacijske energije pri vrhu zareze

This distribution can be characterised by four parameters:

- W^*_{ef} - the effective strain-energy density at the notch tip,
- X_{ef} - the effective distance,
- α' - the slope of the linear part of the strain-energy density distribution,
- W^*_{N} - the net strain-energy density.

The strain-energy density notch intensity factor has been defined in the area of the "pseudo strain-energy density singularities" of the notch in **Figure 5**. This distribution can be considered only for a distance greater than X'_{m}, defined on this figure with the following form:

$$W^* = \frac{K_{\rho,W^*}}{(2\pi r)^{\alpha'}} \quad \text{for } r > X_{\mathrm{ef}} \tag{10}$$

The strain-energy density W^* at the notch tip is a mechanical parameter that can be used as a fracture criterion. For fracture, the average critical strain-energy density W^*_{ef} in the fracture process volume can be used as a local energetic criterion.

Fracture occurs when:

$$W^* = W^*_{\mathrm{ef}} \tag{11}$$

4 LOCAL STRAIN FRACTURE CRITERION

The strain distribution can be presented in a similar way to the stress distribution, in a bilogarithmic graph (**Figure 6**).

Zone III can be assimilated in to a zone of strain pseudo-singularity. In this area the strain-distance relationship has the following form:

$$\varepsilon_{\mathrm{yy}} = \frac{K_{\rho,\varepsilon}}{(2\pi r)^{\alpha''}} \tag{12}$$

A local strain fracture criterion is also based on the concept of fracture volume proceses which has been described in the case of a local stress fracture criterion. The limit of this fracture process is also the beginning of

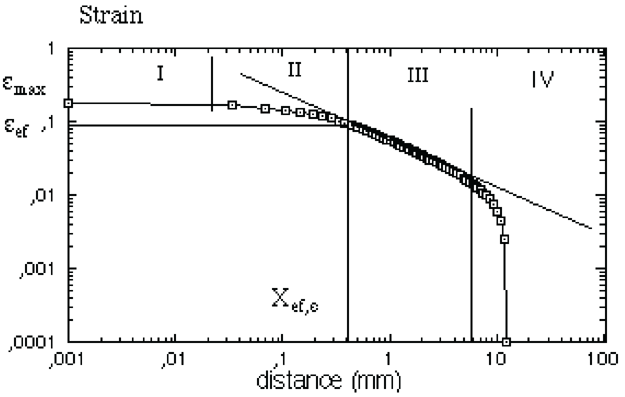

Figure 6 : Example of the elasto-plastic strain distribution at the notch tip presented in a bilogarithmic graph

Slika 6: Primer elastoplastične porazdelitve deformacije pri vrhu zareze v logaritemski obliki

the strain pseudo singularity, which has for abscissa $X_{ef,\varepsilon}$. For the critical event the strain for this abscissa is the critical effective strain $\varepsilon_{ef,c}$.

The product $\varepsilon_{ef,\varepsilon,c}\cdot(2\pi X_{ef,\varepsilon})^{\alpha''}$ is precisely the critical notch strain intensity factor, which can be taken as a measure of the fracture toughness.

$$K_{\rho,\varepsilon,c} = \varepsilon_{ef,\varepsilon,c}\cdot(2\pi X_{ef,\varepsilon})^{\alpha''} \tag{13}$$

In other words, the fracture occurs when the notch strain intensity factor reaches a critical value:

$$K_{\rho,\varepsilon} = K_{\rho,\varepsilon,c} \tag{14}$$

The Notch Ductility Factor (NDF) differs from the critical strain intensity factor by a constant:

$$NDF = \varepsilon^{c}_{ef}(X^{c}_{ef,\varepsilon})^{\alpha''} = \frac{k^{c}_{\rho,\varepsilon}}{(2\pi)^{\alpha''}} \tag{15}$$

An example of the evolution of the notch ductility factor measured on a SENT specimen made of steel is presented in **Figure 7**.

5 CONCLUSION

In notch fracture mechanics a crack is considered as a particular case of a notch with a notch angle and a notch radius equal to zero. To be exact, it is the zero value of the notch radius that leads to a singular distribution for a crack. In reality, stress singularity does not exist because blunting plastificity and damage lead to a stress relaxation and a finite value of the maximum stress. However, the concept of a stress intensity factor which governs the singularity can be kept because at a given distance, the effective distance, the same type of distribution is recovered. In this case we speak about a notch stress intensity factor. The concept can be extended to strain and strain-energy distribution. Its critical value is used as a fracture-toughness parameter. The notch fracture mechanics suggests that the value of the notch stress intensity factor is limited and even erroneus.

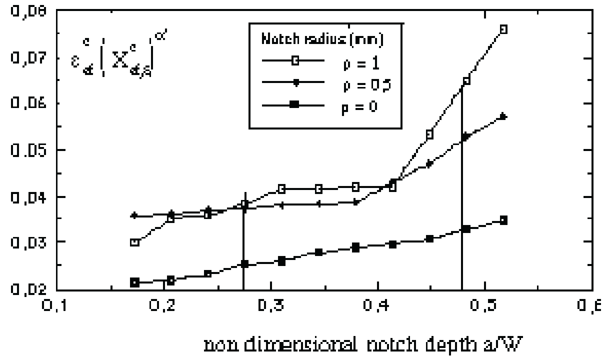

Figure 7: Evolution of the Notch Ductility Factor with a notch radius and a non-dimensional notch depth

Slika 7: Odvisnost med zareznim faktorjem duktilnosti in polmerom zareze ter brezdimenzijsko globino zareze

For a crack, the stress-intensity factor has units of MPa $m^{0,5}$. These units are a consequence of a typical stress gradient at the crack tip. For a notch, if the notch angle is different from zero, the stress gradient is different and leads to other units for the notch stress intensity factor MPa m^{α} with α less than 0.5. This is a difficulty when using this concept because not only the units are changing, but also the fracture toughness increases with notch angle and is not intrinsic to the materials. To overcome this difficulty, the use of a local approach is preferable but in this case the difficulty is to define the fracture process zone.

Nomenclature

σ_{yy}	opening stress
α	parameter governing the stress singularity
α'	slope of the linear part of the strain- energy density distribution
α''	slope of the linear part of the strain distribution
a	notch length
B	thickness
b	ligament size
χ	relative stress gradient
$\varepsilon_{ef,c}$	critical effective strain
G_{Ic}	critical strain-energy release rate
η	parameter of proportionality
J_{IC}	fracture toughness
$J_{Ic,app}$	apparent fracture toughness
K_{ρ}	notch stress intensity factor
$K_{\rho,\varepsilon}$	notch strain intensity factor
$K_{\rho,\varepsilon,c}$	critical notch strain intensity factor
$K_{\rho,W*}$	notch intensity factor for the strain- energy density
R	distance
ρ	notch radius
ρ_{c}	critical notch radius
σ_{ef}	effective distance
$\hat{\sigma}_{ij}$	weighted stress
σ_{ij}	component of the stress distribution
U_{nl}	non-linear work done
$U_{nl,c}$	non-linear work for fracture
W^{*}_{ef}	average critical strain-energy density

W^*_{ef}	effective strain-energy density at notch tip
W^*_N	net strain-energy density
X_{ef}	effective distance
$X_{ef,\varepsilon}$	effective distance for strain distribution

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