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FATIGUE PROBLEMS OF TRANSMISSION BELTS: a viscoelastic analysis of the strain-accumulation process

PROBLEM UTRUJANJA POGONSKIH JERMENOV: viskoelastična analiza procesa akumuliranja deformacije

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We performed an analysis of the time-dependent behaviour of drive belts under the loading conditions to which they are exposed during normal operation. They are dynamically loaded with a tooth-like periodic (cyclic) load. Within each loading cycle the elastomeric material undergoes a combination of creep and retardation processes. Under certain conditions, the retardation process between two loadings cannot be fully completed. Thus, the material enters the second phase of loading with a residual strain state. Consequently, the strain state starts to accumulate, which leads to hardening of the material, crack formation, and ultimately to the failure of the belt.

We recognized that drive belts exhibit the accumulation of strain when exposed to normal operation at certain critical angular velocities. The strain accumulated in each consecutive cycle depends on the geometry of the belt, the angular velocity of the pulleys, the number of completed cycles, and the retardation spectrum of the material. In this paper we discuss the effect of the number of loading cycles to which the material is exposed in the strain-accumulation process. For a given belt geometry the critical angular velocity increases with the number of loading cycles. At the same time the magnitude of the accumulated strain decreases non-linearly as the number of loading cycles increases. Hence, the strain-accumulation process slows down with the increasing number of loading cycles. However, if the belt operates at, or in the close vicinity of, its critical angular velocity, it will almost certainly fail. Since the critical angular velocity is directly related to the retardation time of the material, and the magnitude of the accumulated strain depends on the strength of the corresponding discrete spectrum lines, we can conclude that the time-dependent mechanical properties of the elastomersic material from which the belt is constructed are the most critical parameters for predicting the durability of drive belts and other dynamically loaded elastomeric products.

Key words: time-dependent constitutive modelling, power transmission belts, synchronous belts, failure, viscoelastic analysis, strain accumulation, elastomers, time-dependent behaviour, mechanical spectrum

V delu predstavljamo viskoelastično analizo časovno odvisnega vedenja pogonskih jermenov, ki so med obratovanjem motorja dinamično obremenjeni s koračno spremembo. V času enega obremenitvenega cikla je posamezna viskoelastična komponenta jermena izpostavljena kombiniranemu procesu lezenja in relaksacije. Pri določenih pogojih, ki jih opredeljujeta geometrija jermena in kotna hitrost jermenice, se proces retardacije med dvema obremenitvenima cikloma ne zaključi. To pomeni, da material vstopi v naslednjo obremenitveno fazo s preostalo deformacijo. Postopoma se začne deformacija akumulirati, kar vodi do utrjevanja materiala, nastanka razpoke in posledično do propada jermena.

Analiza pokaže, da se akumulacija deformacije v jermenu pojavi pri določenih kritičnih vrednostih kotne hitrosti. Proces akumuliranja deformacije v vsakem zaporednem obremenitvenem ciklu je odvisen od geometrije jermena, kotne hitrosti jermenice, števila zaključenih obremenitvenih ciklov in retardacijskega spektra materiala, iz katerega je narejen jermen. Predstavljeno delo obravnava vpliv števila obremenitvenih ciklov na proces akumuliranja deformacije v pogonskem jermenu med obratovanjem motorja. Pri dani geometriji jermena kritična kotna hitrost narašča s povečevanjem števila obremenitvenih ciklov, medtem ko velikost akumulirane deformacije nelinearno pojema, kar kaže, da se z naraščanjem števila ciklov obremenjevanja proces akumuliranja deformacije upočasni. Rezultati kažejo, da bo jermen med obratovanjem pri kritični kotni hitrosti (oz. v njeni bližini) zelo verjetno odpovedal. Z ozirom na to, da je kritična kotna hitrost povezana z retardacijskim časom materiala in je velikost akumulirane deformacije odvisna od jakosti pripadajoče diskretne spektralne linije, lahko sklepamo, da so časovno odvisne mehanske lastnosti elastomernega materiala, iz katerega je narejen jermen, kritični parameter za napoved trajnosti jermenov in drugih dinamično obremenjenih elastomernih produktov.

Ključne besede: časovno odvisno konstitutivno modeliranje, pogonski jermeni, utrujanje, propad jermena, viskoelastična analiza, akumulacija deformacije, elastomeri, časovno odvisno vedenje, mehanski spekter

1 THE LOADING CONDITIONS OF A SYNCHRONOUS BELT

The loading conditions of a synchronous belt were determined with the commercial FEM program called ANSYS, assuming the elastic behaviour of all belt components. The belt was pre-stressed with a force F and loaded with a desired torque M so that the pre-stressing force was adequately divided into the strand on the tension side, F_1, and the strand on the slack side F_2, as schematically shown in **Figure 1**. The distance between the two pulleys is indicated as l, the radius of both pulleys is R, and the times t_1 and t_2 indicate when the belt enters and leaves the driving pulley.

The driving pulley and the driven pulley were then rotated such that a selected point on the belt would return to its initial position. Such a movement we designated as a complete loading cycle of the belt. For a further analysis of the strain accumulation, we selected the location on the tooth where the shear stress state within the loading cycle, calculated with the FEM program, has the form of an impulse function. This location is indicated as point A in **Figure 2a**. The corresponding

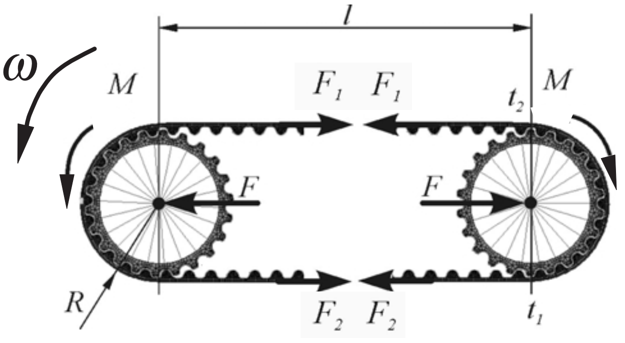

Figure 1: Schematic diagram of the belt-loading conditions and the geometry
Slika 1: Shematičen prikaz obremenjevanja jermena in njegove geometrije

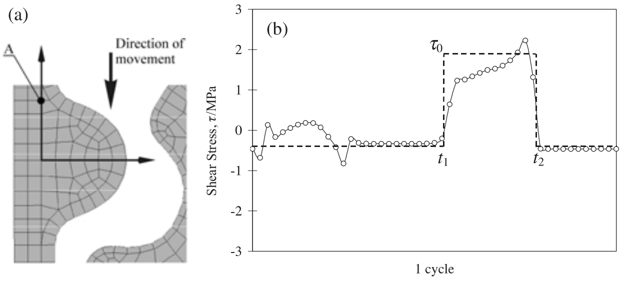

Figure 2: Shear stresses at point A within a complete loading cycle
Slika 2: Spreminjanje strižne napetosti v točki A v enem obremenitvenem ciklu

calculated time-dependent evolution of the stress state is shown in **Figure 2b**.

As a first approximation the shear stress may be modelled as the difference between two step functions with the shear stress intensity τ_0. Hence,

$$\tau(t) = \tau(0)h(t) + \\ + \tau_0 \sum_{n=1}^{n=N} \{h[t - t_1 - (n-1)\xi] - h[t - t_2 - (n-1)\xi]\} \quad (1)$$

The times t_1 and t_2 and the duration of one loading cycle ξ are functions of the geometry and the angular velocity of the belt drive, ω. Here, N is the number of cycles to which the belt has been exposed, and $\tau(0)$ is the shear stress at $t = 0$. Hence, $t_1 = (l+\pi R)/(\omega R)$, $t_2 = (l+2\pi R)/(\omega R)$, and $\xi = (2l+\pi R)/(\omega R)$.

2 ANALYSIS OF THE STRAIN ACCUMULATION

The time-dependent strain response of a rubber material can be expressed as [1],

$$\gamma(t) = \tau(0)J(t) + \int_0^t J(t-s)\frac{\partial \tau(s)}{\partial s}\mathrm{d}s \quad (2)$$

where $J(t)$ is the shear creep compliance. Assuming the material is simple and can be modelled with a single spectrum line, the material function is expressed as $J(t) = J_g + L_1 e(-t/\lambda_1)$, where J_g denotes the glassy compliance and λ_1 is the retardation time where the corresponding spectrum line $L_1 = (\lambda_1)$ is located.

Let us analyse the accumulated strain at the end of N completed cycles at $t = t_N = N\xi$. Substituting Equation (1) into Equation (2), and considering the geometry of the drive belt as $l/R = \pi$, we obtain

$$\gamma(N) = \tau(0)J\left(\frac{4\pi N}{\omega}\right) + \Gamma_N(N),\ \Gamma_N(N) = \sum_{n=1}^{N} \Delta\Gamma_n(n) \quad (3)$$

where

$$\Delta\Gamma_n(n) = \tau_0 L_1 \exp\left(\frac{\pi(4n-3)}{\omega\lambda_1}\right)\left[1 - \exp\left(-\frac{\pi}{\omega\lambda_1}\right)\right] \quad (4)$$

denotes the strain accumulation for each consecutive cycle. Equation (3) describes the time-dependent evolution of the strain field in the material when exposed to cyclic loading. **Figure 3** shows the strain accumulation in each consecutive cycle, expressed by Equation (4), as a function of lg ω for different numbers of completed cycles. From **Figure 3** it is clear that the strain accumulation has its maximum at a certain critical angular velocity.

The location of the critical angular velocity, $\omega_{CR}(n)$, can be obtained by equating the two last terms in Equation (4),

$$1 - \exp(-\pi/\omega_{CR}(n)\lambda_1) = \exp[\pi(4n-3)/\omega_{CR}(n)\lambda_1] \quad (5)$$

The critical angular velocity at which the accumulated strain has a peak magnitude is a close form solution of Equation (5) and can be expressed as

$$\omega_{CR}(n) = \frac{\pi}{\lambda_1 \ln \dfrac{4n-2}{4n-3}} \quad (6)$$

It is important to note that the critical angular velocity is directly related to the retardation time of the

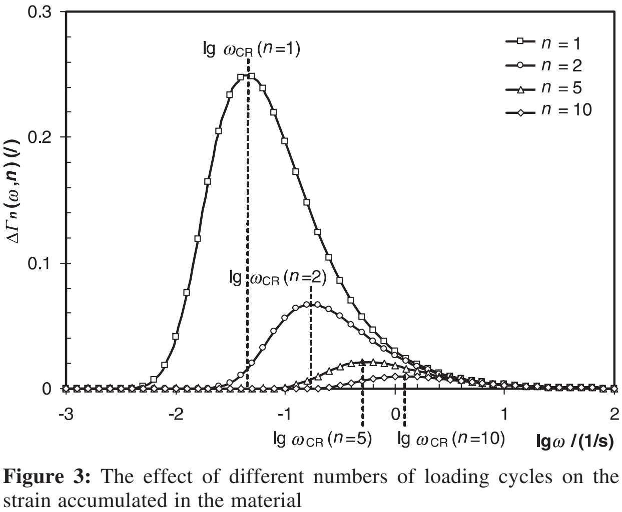

Figure 3: The effect of different numbers of loading cycles on the strain accumulated in the material
Slika 3: Vpliv števila obremenitvenih ciklov na akumulacijo deformacije v materialu

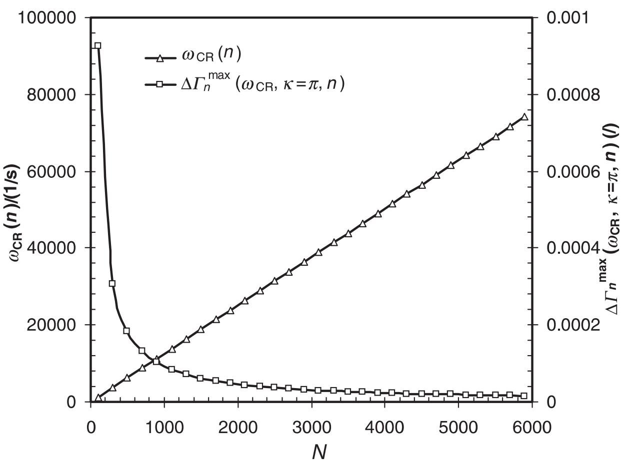

Figure 4: The critical angular velocity and the maximum strain accumulation as a function of the number of loading cycles

Slika 4: Kritična kotna hitrost in največja akumulirana deformacije v odvisnosti od števila obremenitvenih ciklov

material, which clearly emphasizes the importance of the material's time-dependent properties. By combining Equations (3) and (6) we obtain the corresponding peak value of the accumulated strain in each loading cycle,

$$\Delta\Gamma_n^{\max}(\omega_{CR}, n) = \tau_0 L_1 \frac{1}{4n-2}\left(\frac{4n-3}{4n-2}\right)^{(4n-3)}; \quad n = 1, 2, 3 \ldots, N \tag{7}$$

The critical angular velocity and the corresponding peak value of the accumulated strain in each loading cycle as a function of n for λ_1 = 100 s^{2} are shown in **Figure 4**. From the figure we can clearly see that the critical angular velocity shifts towards higher frequencies for each consecutive loading cycle, and that the relation is almost linear. At the same time the corresponding accumulated strain decreases for each consecutive cycle.

Providing the drive belt can operate at the critical angular velocity at all times, the maximum strain that could be accumulated over N cycles would be,

$$\Gamma_n^{\max}(\omega_{CR}, n) = \sum_{n=1}^{n=N} \Delta\Gamma_n^{\max}(\omega_{CR}, n) = \tau_0 L_1 \sum_{n=1}^{n=N} \frac{1}{4n-2}\left(\frac{4n-3}{4n-2}\right)^{(4n-3)} \tag{8}$$

It is important to stress that

$$\lim_{n\to\infty} \Delta\Gamma_n^{\max}(\omega_{CR}, \kappa=\pi, n) = \lim_{n\to\infty} \tau_0 L_1 \frac{1}{4n-2}\left(\frac{4n-3}{4n-2}\right)^{(4n-3)} = 0 \tag{9}$$

which means that the strain accumulated during each cycle will tend towards zero value as $n \rightarrow \propto$. However

$$\lim_{n\to\infty} \Delta\Gamma_n^{\max}(\omega_{CR}, N) = \lim_{n\to\infty} \tau_0 L_1 \sum_{n=1}^{n=N} \frac{1}{4n-2}\left(\frac{4n-3}{4n-2}\right)^{(4n-3)} = \propto \tag{10}$$

which means that if a belt were to operate all the time at the critical angular velocity it would always fail.

The total accumulated strain, expressed by Equation (8), as function of the cumulative number of loading cycles is shown in **Figure 5**.

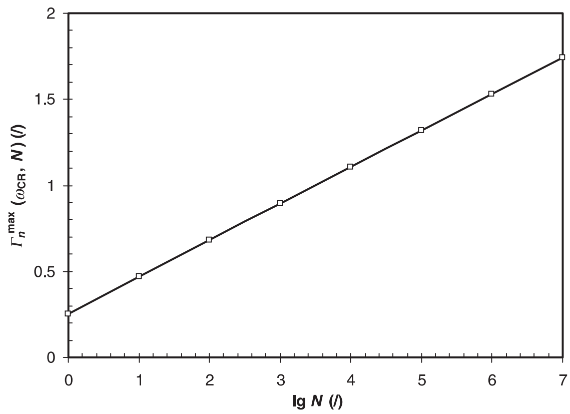

Figure 5: The total accumulated strain as a function of the cumulative number of loading cycles

Slika 5: Celotna akumulirana deformacija v odvisnosti od kumulativnega števila obremenitvenih ciklov

3 CONCLUSIONS

From the presented analysis we can conclude that drive belts will exhibit the accumulation of strain when exposed to normal operation at certain angular velocities. For a given belt geometry the critical angular velocity increases with the number of loading cycles. At the same time the magnitude of the accumulated strain decreases non-linearly as the number of loading cycles increases. Hence, the strain-accumulation process slows down with the increasing number of loading cycles, and will be negligible after a certain number of loading cycles, as shown in **Figure 4**.

However, if the belt operates at, or in the close vicinity of, its critical angular velocity, it will almost certainly fail. The critical angular velocity depends on the material's retardation time (i.e., the location of the mechanical spectrum), while the magnitude of the accumulated strain depends on the strength of the corresponding discrete spectrum lines. Thus, the time-dependent mechanical properties of the elastomeric material from which the belt is constructed are the most critical parameters for predicting the durability of drive belts and other dynamically loaded elastomeric products.

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