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ON THE DETERMINATION OF SAFETY FACTORS FOR MACHINES USING FINITE ELEMENT COMPUTATIONS

O DOLOČITVI FAKTORJEV VARNOSTI ZA NAPRAVE PRI IZRAČUNU Z METODO KONČNIH ELEMENTOV

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Principles for the selection of modern methods for the determination of local strength safety factors in design computations of GTE parts for static and cyclic loading are suggested. It is shown that the selection of methods for the evaluation of local strength safety factors should be carried out applying special criteria and computations including adequate models of visco-elasto-plasticity. On the basis of the analysis of computational practice the minimum values of local strength safety factors for static and cyclic loading, which may be recommended for FEM computations, have been proposed.

Key words: safety factor, finite element computation, creep, loading, cycle, fatigue

Predloženi so principi za izbiro metod za določitev lokalnih faktorjev varnosti za GTE-dele in za statično ter ciklično obremenitev. Izbira metod za oceno lokalnega faktorja varnosti za trdnost je mogoča z uporabo primernih meril in primernih modelov visko-elasto-plastičnosti. Na podlagi analize prakse izračunavanja so predložene najnižje vrednosti za faktorje varnosti za lokalno trdnost za statično in ciklično obremenitev pri FEM-izračunih.

Ključne besede: faktor varnosti, izračuni z metodo končnih elementov, lezenje, obremenitev, cikel

1 INTRODUCTION

The wide and universal propagation of commercial finite element packages (**ANSYS, ABAQUS, MARC, LS DYNA, NASTRAN** etc) for computations in design of machines and civil structures made possible to define more accurately the stress-strain state (SSS) and opened the way to solve some problems connected with the normalization of safety factors. One of these problems is the determination of the possibility of defining more reliable values for safety factors values based on the more accurate knowledge of the SSS of the construction. The more reliable reduction of safety factors would allow to decrease the weight of material for the construction, however, it may also increase the risk of flaws arising during the exploitation. The application of the finite element methods (FEM) for computing SSS in the locations of stress concentration makes it possible to design more accurately the configuration of these details of components, to obtain minimal stresses, increase the life time of parts supporting static stresses, and, more importantly, also the lifetime of parts submitted to cyclic loading. FEM is indispensable for prognosticating the crack propagation rate in parts with geometry, temperature and stress gradients where conventional computation schemes cannot be applied with sufficient reliability.

In the process of normalization of strength values for parts, it should be provided for the introduction of safety factors, both for material properties and for loading parameters of a construction. In both cases the risk may arise of use of non verified data.

It is known that some cases damages of parts during the turbine exploitation were caused by the improper evaluation of local strength in the stage of design. As examples of such events may be mentioned in particular cracks networks revealed at the inspection of gas turbine rotors after a determined operational time; cracks on gas turbine disk rims; thermal fatigue cracks on the border and the back of cooled working and regulation blades and cracks on components of combustion chambers of gas turbine engines (GTE).

The basic principles for the normalizing of safety factors considering the local static and low-cycle fatigue strength in this paper were experimentally verified in an independent report. The experience of normalizing safety factors, both in gas turbine and reactor design is widely used [1,2,3].

The application of FEM requires a high qualification designer skilled in computational mechanics, inasmuch as the computation results depend significantly on the methods of partitioning an analyzed part with a finite element (FE) network and the selection of FE types. With the aim to describe the material properties in determining the SSS of constructions applying FEM, the method of "average values minus two or three average quadratic deviations" is used. In some cases, for example, for creep strain, it is necessary to apply "average values plus two or three average quadratic deviations". However, the insufficient quantity of experimental data for new materials, insufficient information's on the dependence of properties on operating conditions as well as data accounting for the influence of environment, restricts the application of this method. In this situation, to make easier the proper application of available experimental data, is expedient to use a sufficiently widespread concept of the "upper and lower envelop curves"[4].

Generally, the process of rupture at static loading may be of three types:

a) exhaustion of short-time plasticity,
b) creep,
c) brittle.

It is evident that the differentiation of safety factors depends on the type of rupture and should be considered in the normalization of local stresses. It is clear that the greatest safety factor value should be considered in the case of brittle fracture that may occur in the range of maximal scatter of material parameters.

2 STATIC STRENGTH

2.1 Static strength of deformable materials

The presence of stress concentration does not lead to a decrease of the bearing capacity of deformable materials in case of short-time or long-time static loading. From here on, the term "bearing capacity of plastic materials" should be understood as the conditions in which the ultimate load causing the rupture of a construction is determined with the loss of bearing capacity according to the "plastic hinge mechanism". If the value of long-time plasticity of a deformed material exceeds 4–5 %, it is not sensitive to the notch effect in long-time strength tests. Also heterogeneous cast alloys are not notch-sensitive. The temperature dependence of the plasticity of materials is not monotonous. Thus, analyzing a material state with consideration of the exploitation parameters, it is necessary to have on disposal the data of material deformability as function of the temperature and the strain rate (creep rate).

The analysis of experimental and calculated data indicates that the value of the ratio $K^{\sigma} = \sigma_B^{n}/\sigma_B^{s}$ may be taken as a criterion for the material plasticity (σ_B^{n} and σ_B^{s} are ultimate strength values determined by testing notched and smooth specimens). Alloys with $K^{\sigma} \geq 1.3$ obtained at appropriate temperatures for specimens with $\alpha_{\sigma} = 3.5$–4.5 (α_{σ} is coefficient concentration of stress), submitted to short and long-time tensile tests, are not propensive to brittle rupture. The use of alloys with $K^{\sigma} < 1.3$ is permitted only on the base of results of appropriate tests that include the statistical evaluation of results of tests of specimens with initial cracks (Sharpy impact tests) and low-cycle fatigue characteristics obtained from tests with notched specimens. We may assert, for this reason, that the introduction of FEM-computations for parts from plastic materials and the more precise determination of SSS at stress concentration locations should not be the base for the correction of safety factors related to the bearing capacity of constructions. At the same time, if the bearing capacity of constructions is ensured, the assumption of the needlessness of evaluation of safety factors related to the static strength and based on local stress values, is justified.

The analysis of the criteria defined in the strength standards [1,2], as well as the suggested approaches to the evaluation of the static strength and the experience of exploitation of various parts show that all attempts to restrict the value of yield strength are senseless. On the other hand, it became generally accepted that in case of appropriate ultimate strain exceeding 4–6 %, it is not necessary to take into account the residual stresses in the computations of static strength. The same is valid also for the thermal stress σ_T, if $\sigma_T = 2\alpha\Delta T < 2E$ % (α – coefficient of linear expansion; E – Young elastic modulus; ΔT – range of temperature variation).

In such approach to the normalization of the static strength of constructions, it is necessary to verify the respect of the condition that the value of J-integral is below its critical value J_c. Thus, for example, according to [1], in this case the maximal nominal static stresses (without accounting for concentrators) for pressure vessels are permitted to be below of 1/1.5 for yield strength and below of 1/2.6 for tensile strength.

The following approaches are expedient to apply for the evaluation of safety factors related to local stresses:

1. The application of the proper model of kinematical hardening is justified for solving many practical problems. However, the optimal is the SSS computation and the choice of a plasticity model depend on the material analyzed and of the loading in accordance with the conception of multimodel approach[8].
2. The static strength of deformable materials should be evaluated on the base of exhaustion of the ultimate material plasticity ε^*, which, in turn, depends on the loading rate or time. Incidentally, one should differ ultimate states for intragranular and intergranular rupture. Intragranular rupture is characterized by the absence of dependence of ultimate strains on loading rate, at the same time, for intragranular rupture, the ultimate strain diminishes with the decrease of loading rate.
3. If the local strength is evaluated with respect to the short-time plastic strain, the safety factor on strains $\varepsilon^*/\varepsilon_p$ (ε_p – plastic strain) should not be lower than 2.0, with ε^* defined with regard to the stress state triaxility by the following equations

$$\varepsilon^* = \varepsilon_p^{ult}\, 1.7\exp(-1.5\bar{\sigma}/\sigma_i) \tag{1a}$$

$$\varepsilon^* = \varepsilon_p^{np}\, K_e \sigma_i^2/3(\sigma_i \sigma_{cp}) \tag{1b}$$

which give a conservative estimation of the plasticity. Here

ε_p^{ult} – ultimate strain (deformability) at short time tension;

K_e – characteristic of material state (at brittle state – $K_e = 1$, at plastic state – $K_e = 1.2$);

$\bar{\sigma}$ – mean stress.

The value of ε_p is defined with elastoplastic computation using an appropriate plasticity model and the lower strain envelop curve. In this case safety factor on stresses shall not be lower than 1.2–1.4.

It should be noted also, that the problem of normalizing of the static strength needs further development on the base of comprehensive investigations of material properties aimed to the improve the plasticity models for computing three-dimensional SSS and to further develop the rupture criteria. It should be noted that it is necessary to adapt effectively, after comprehensive testing, new plasticity models to commercial FE packages.

2.2 Safety factors for local strength for creep loading

By considering the safety factors for local strength, it is expedient to proceed from the following considerations:

1. The evaluation of rupture situation of parts operating at creep deformation can be implemented with applying the ultimate strain value, which depends on temperature, time and of the stress state rigidity. Therefore, as in the case of normalizing, the safety factors for static strength of parts from deformable materials, the use of FEM computations and the more exact knowledge of SSS for stress concentration locations cannot be the base for correcting the values of creep safety factors. In this case, there is no need to use of modern methods for stress computation. The safety factors for creep should be defined with applying the crack initiation criteria.
2. Correction of modern safety factors for creep should be based on the improvement of creep models, especially applied to parts operating in three-dimensional stress state and submitted to multifactor and nonstationary loading, as well as on the results of the analysis of creep characteristics and long-time strength of materials.
3. For the description of the influence of material properties and stress complexity in a part on its deformability, it is expedient to apply the following equations that are analogous to (1a) and (1b)

$$p^* = 1.7\ \varepsilon_c \exp(-1.5\bar{\sigma}/\sigma_i) \tag{2a}$$

$$p^* = \varepsilon_n K_e \sigma_i^2 / 3(\sigma_i \sigma_{cp}) \tag{2b}$$

where:

ε_c – critical creep strain at uniaxial loading;

p^* – ultimate creep strain (deformability) at the complex stress state;

K_e – characteristic of material state ($K_e = 1$ – for brittle state and $K_e = 1.2$ – for plastic state).

These, as well as equations (1a) and (1b), give a conservative evaluation of the ultimate strain. In this case and considering the values of accumulated creep strains along the upper envelop curve, the minimal strain safety factor value should not be below 2. For the determination of the safety factors on life time ($K_{\tau,N}$) and on stresses (K_σ), it is recommended to use the life-time lower envelop curves obtained with the probability of 99 %. It is expedient to apply safety factor values not lower than $K_\sigma = 1.2$ and $K_{\tau,N} = 1.5$. In this range the minimal value of the safety factor should be selected. In some situations the values of safety factor may be determined with the use of the average curves and depending on the scatter of material properties, the safety factors should be not less than $K_\sigma = 2$ and $K_{\tau,N} = 10$.

4. It has been shown in several investigations (STP ASTM No 165, 1954 [11]) that for the accounting of a nonstationary situation in computations using the formulas of linear summation of damages (in deformation or time interpretation), a conservative estimation is obtained on condition that the sum of damages is taken equal to 0.87. For the constructions submitted to a large number of launchings and stops it is necessary to take into account the effect of cyclic loading on the parameters of creep and life-time strength.

2.3 Safety factors in conditions of brittle fracture

The criterion characterizing the brittle fracture is the value of plain strain stress intensity factor K_1. For constructions with flaws, the computed values of stress intensity factor K_1 should be compared with its critical K_{1c} value. The brittle strength is assumed to be ensuring if the following condition is observed:

$$K_1 \leq K_{1c} \tag{3}$$

It is recommended to calculate the value of K_1 according the following equation (1):

$$K_1 = \eta \cdot \frac{(\sigma_p M_p + \sigma_q M_q)\cdot(\pi a/10^3)^{0.5}}{\left[1 + 4.6(a/2c)^{1.65}\right]^{0.5}} \tag{4}$$

where:

η – coefficient accounting for the influence of stress concentrations;

σ_p – tension component of stress intensity;

σ_q – bending component of stress intensity;

$M_p = 1 + 0.12(1 - a/c)$; $M_q = 1 - 0.64\ a/h$;

a – crack depth, generally assumed to be elliptical;

c – crack half length;

h – area within which bending stress component remains positive (the value of h for the formula (4) is permitted to be taken equal to half wall thickness).

For constructions with non detected flaws, the value of K_1 (according[1]) should be computed assuming the presence of defects of size comparable with the sensitivity of the inspection apparatus. Here, it is also necessary to account for the dimensions of a "shaded" zone where it is impossible to check up the presence of flaws in exploitation. In design, it is generally assumed that the construction should ensure the safety for the crack of size equal to 1/4 thickness of the part bearing section (wall), that is considered as defect size in the computation[3]. In case of sufficiently careful inspection during the exploitation, the crack size may be taken as equal to the sensitivity parameter of the inspection apparatus, or in case of detected crack, to the size of the crack. As a rule, it is assumed that the safety factor on K_{1c} shall be not less than 2 and a lower safety factor may be adopted in case of availability of sufficient statistical data.

3 CYCLIC STRENGTH

In the case of evaluation of cyclic strength, various methods have been suggested for determining the local strength safety factors. These may be conditionally divided in five groups: computational for a rigid cycle, computational for a general situation, computational-experimental, based on the theory of adaptability and based on deformation criteriia.

3.1 *For cyclic loading and rigid cycle* (case of uniaxial loading with cycle asymmetry coefficient $r \approx -1$), it is expedient to use the values of amplitude intensity of conditionally elastic full strains as parameters of loading:

$$\Delta\varepsilon = \sqrt{2/3\Delta\varepsilon_{ij}\varepsilon_{ij}} \quad (5)$$

The resistance to fatigue for elastic cyclic deformation is evaluated applying the Goodman's equation:

$$\sigma_{\max} = \sigma_{-1}(1-\overline{\sigma}/\sigma_B) \quad (6)$$

with:

$\sigma_{\max}$ – maximum cycle local stress with account of stress concentration;

$\overline{\sigma}$ – average cycle stress;

σ_{-1} – endurance limit of a material for symmetrical cycle with account of stress concentration.

The determination of the resistance to elasto-plastic deformation at cyclic loading is possible using of cyclic strain curves. In this case the conditions for the rupture at elasto-plastic cyclic deformation is obtained applying the Coffin's deformation criterion:

$$\sum (\Delta\varepsilon_p^{(k)}/\varepsilon^*)^m \, \mathrm{d}N = 1 \quad (7)$$

where:

$\Delta\varepsilon_p^{(k)}$ – plastic strain amplitude in K-cycle;

m – constant;

ε^* – ultimate strain deduced applying the equations (1a) and (1b).

In connection with the generally insufficiency of experimental data for the statistical analysis, as in case of creep static loading, it is expedient to evaluate the value of local strength safety factor with the use of the average curve and assuming the value of 2 in case of evaluation based on stress (or strain) amplitude, or equal to 10 if the evaluation is based on life time parameters. However, if the quantity of experimental data is sufficient and the lower envelop curve is reliable, the values of safety factors defined by stresses and by life time parameters may be taken equal to 1.2 and 1.5, correspondingly. It should be noted that the values of deformability ε^* and coefficient m in equation (6) should be determined experimentally for every material. In the case of using the universal value of coefficient m in Coffin's equation (7), the values of safety factors should be increased.

3.2 *For cyclic loading in a general case*, when the unilateral accumulation of strain (characteristic for a mild cycle and generally called "ratcheting") and stress variation (characteristic for a rigid cycle) takes place, different approaches to the evaluation of cyclic strength safety factors may be applied. Among all the known strength characteristics of a material, the life time under cyclic loading is depends mostly on the influence of factors related to the construction, technology, metallurgy and operation. Therefore, the evaluation of the life time under cyclic loading for constructions is possible considering the results of test specimens and construction components with accounting of all above mentioned factors. The main operational factors affecting the life time of a part under cyclic loading, are temperature and holding time at maximal loads and temperature, cyclic asymmetry, superposition of high-frequency component upon the low-frequency variation of loading. The realization of tests within all the range of operational loading is a rather labor consuming task. Therefore, is quite urgent to develope methods based on conventional tests of specimens for the evaluation of life time of constructions submitted in operation to complex loading.

For low-cyclic loading, material damages may be computed applying the deformation or energy criteria of rupture. Here, for computing the kinetics of stress-strain state, both for complex noncyclic loading and for cyclic loading with altering loading parameters, instead of a number of cycles n (or number of semi cycles k) it is expedient to use the relations of Odquist's type as parameters of of the actual state of the material. These relations are expressed by the following formulas [12,13]:

$$\lambda_1 = \int \mathrm{d}\varepsilon_{\mathrm{p}} - \varepsilon_{\mathrm{p}}; \; \mathrm{d}\varepsilon_{\mathrm{p}} = (2/3\mathrm{d}\varepsilon_{\mathrm{p}ij}\mathrm{d}\varepsilon_{\mathrm{p}ij})^{0.5};$$
$$\varepsilon_{\mathrm{p}} = (2/3\mathrm{d}\varepsilon_{\mathrm{p}ij}\mathrm{d}\varepsilon_{\mathrm{p}ij})^{0.5} \quad (8a)$$
$$\lambda_2 = \int \mathrm{d}p - p; \; \mathrm{d}p = (2/3p_{ij}p_{ij})^{0.5};$$
$$p = (2/3p_{\mathrm{p}ij}p_{\mathrm{p}ij})^{0.5} \quad (8b)$$
$$\Delta\lambda_1 = \lambda^{(k)} - \lambda^{(k-1)} \ge 0$$

k – the ordinal number of a semi cycle.

The increment of nonelastic strains ($\mathrm{d}\varepsilon^{\mathrm{ne}}$) and the value of nonelastic strain intensity ($\varepsilon^{\mathrm{ne}}$) are defined with the equations:

$$\mathrm{d}\varepsilon_{ij}^{\mathrm{ne}} = \mathrm{d}\varepsilon_{\mathrm{p}ij}\mathrm{d}p_{\mathrm{p}ij}$$
$$\varepsilon^{\mathrm{ne}} = (2/3\mathrm{d}\varepsilon_{ij}^{\mathrm{ne}}\mathrm{d}\varepsilon^{\mathrm{ne}})^{0.5} \quad (9)$$

For the case of creep for known stress, the accumulated creep strains should be distinguished from the nonelastic strains.

In the particular cases of cyclic loading, instead of the mentioned parameters, by simple transformation the computation of damages is replaced with the traditional applying the values of cycles and semi cycles. Then, for the evaluation of the life time under cyclic loading, Coffin's type formulas may be used:

$$(\Delta\varepsilon_{\mathrm{p}})^2 N = C_1 ; \sum \Delta\varepsilon_{pi}^2 = C_1 \qquad (10a)$$

$$(\Delta p)^n N = C_2 ; \sum (\Delta p_i)^n = C_2 \qquad (10b)$$

For the evaluation of the static component of life time, the value of ε_{p} (or Odquist's parameter) is compared with the ultimate strain ε^* of a material.

The described approach probably has one only deficiency, it is unsuitable for characterizing damages in conditions of neutral loading ways (neutral load path). This deficiency in the describing of the nonstationary cyclic loading can be avoided by applying the Coffin's formula (10a) and V. V. Novozhilov's suggestion[16] on the dependence of the accumulation rate of micro damages p:

$$D = k\int \rho \mathrm{d}\lambda = A; \; p = G\varepsilon_{\mathrm{p}} \qquad (11)$$

where $G = \mathrm{d}\sigma/\mathrm{d}\varepsilon_{\mathrm{p}}$ is deformation hardening parameter.

3.3 *The methods of adaptability computation* allow to determine the cyclic strength safety factors for the general case for sign-variable flow and increasing deformation[10,11]. The ultimate material characteristics for the sign-variable flow are:

σ_{s} – half value of the cyclic yield strength $S_{0.4}$ in a stable cycle with the tolerance of plastic strain amplitude within the cycle of 0.4 %.

In the case of presence of stress concentrators:

$\sigma_{\mathrm{s}} = \sqrt{E\varepsilon(N)\sigma_{(\varepsilon N)}}$, with $\varepsilon(N)$ – semi amplitude of the full strain corresponding to the appearance of low-cycle fatigue macro crack in N cycles and

$\sigma_{\varepsilon(N)}$ – in accordance with $\varepsilon(N)$ on the isochronous cyclic strain curve.

For creep in one semi cycles: $\sigma_{\mathrm{s}} = S_{0.4}^{\mathrm{c}} - 0.5S_{0.4}$, with $S_{0.4}^{\mathrm{c}}$ – cyclic yield strength by presence of creep.

For the progressing deformation, as ultimate characteristics $\sigma_{\mathrm{s}} = \sigma_{\mathrm{B}}$ – for transitional modes and $\sigma_{\mathrm{s}} = \sigma_{\mathrm{LTS}}$ $(t,\Sigma\Delta\tau)$ – for stationary modes are taken with σ_{LTS} – long-time strength in accordance with the all life time length of loading.

In [16,17,18] the results of the analyses of stress-strain state and strength of the disks and rims of regulating apparatus (two- and three-dimensional computations) are discussed. The following values of safety factors may be recommended for the strength computations of GTE disks : K_{SVF} = 1.2–1.5 – for sign-variable flow and K_{PD} = 1.9–2.2 – for progressing deformation. For the central part of disks is preferable to specify higher values of safety factor K_{SVF} and lower K_{SVF} values for not central stress concentrators.

4 CONCLUSIONS

1. Principles for the selection of methods for the determination of local strength safety factors in design computations of GTE parts for static and cyclic loading have been proposed. The methods are to be used in solutions of edge problems applying digital-analytical methods, e.g. FEM.
2. It has been shown that the selection of methods for the evaluation of local strength safety factors should be carried out applying special criteria and computations including adequate models of visco-elasto- plasticity.
3. It has been shown that the attempts to limit the values of local static stresses by the value of yield strength are without sense.
4. On the basis of the analysis of computational practice the minimum values of local strength safety factors for static and cyclic loading, which may be recommended for FEM computations, have been defined.

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